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ABSTRACT

A trend analysis statistical package and additional programs for the Apple microcomputer are presented. They illustrate strategies of data analysis suitable to the graphics and processing capabilities of the microcomputer. The programs analyze data sets using examples of: (1) analysis of variance with multiple linear regression; (2) exponential and higher order regression; (3) orthogonal tests of trend; and (4) the use of dummy variables to handle categorical variables. Microcomputer benefits and limitations are highlighted with the exploratory analysis techniques. An example of a traditional analysis of variance test is presented which illustrates the microcomputer data entry procedure. Graphic displays of program statements and diagrams are included with an appendix of sample programs. (CM)

TREND ANALYSIS USING MICROCOMPUTERS

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Presented at the
Annual Meeting of the
National Association for Research
in Science Teaching

April 1981

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-TREND STATISTICAL PACKAGE-

STATISTICAL PROGRAMS

FOR THE APPLE BY

CARL F. BERGER
UNIVERSITY OF MICHIGAN

]

DISK VOLUME 254

*A 012 ANOVAKP
*B 014 AS CHR GEN
*B 006 CHARACTER TABLE
*B 014 COUNT
*A 009 EDIT
*A 013 EXPOREGRESS
*A 016 FNXGRAPH/PADDLES
*A 012 GRAPH RESULTS
*A 002 HELLO
*B 003 HI-RES CHARACTER GENERATOR
*A 011 HLO
*B 002 LOMEM:
*A 012 MLRKP
*A 018 MLRTWOWAYKP
*A 015 NTH-COR
*A 016 NTH-CORKP
*T 002 THLO
*A 004 TITLE
*A 019 TRENDKP
*A 014 TWOWAYKP
 A 002 DATA
 A 013 EXPOREGRESSKP
*R 019 APA
*A 003 LOADAPA
*B 003 RBOOT
*B 005 RLOAD

Looking for relationships is a natural role of science education. More and more it has become important not only to test for differences in groups but also to analyze data for trends. Finding relationships in data sets gives more information than traditional tests of mean differences or traditional ANOVA tests or even MANOVA tests. The use of techniques such as Multiple Linear Regression, Higher Order Correlation, and Exponential Regression can often indicate relationships too easily overlooked in traditional research.

Trend analysis has not been used widely, partly because of the need for extra information added to existing data in order to search for trends. Further, the reliance on numerical values rather than graphs to indicate trend has not provided the necessary comfort needed to explore data.

The microcomputer has capabilities which overcome these drawbacks. First, the graphics mode of the television screen offers greater possibilities in looking for trends. Second, the ability to easily adapt data sets encourages the search for trends. Third, the microcomputer can be used to compare several kinds of analyses in order to get a real feel for the structure and implications inherent in the data.

In this session, the microcomputer will be used to:

1. Compare alternate strategies of data analysis.
2. Graphically show results of tests for trend.

3. Indicate the powerful application of such machines for reasonable size data sets.

Analysis of data sets will be examined using a comparison of:

analysis of variance with Multiple Linear Regression;

the use of Exponential Regression and Higher Order Regression;

the use of orthogonal tests of trend; and

the use of dummy variables to handle categorical variables.

Throughout the session the use of exploratory analysis techniques will be emphasized. The limitations as well as the benefits of the use of microcomputers will be highlighted.

Sample programs are available in the appendix; printouts of t.v. screen data are included. The examples used are from Kerlinger and Pedhazur,[1] with specific page references included.

The first analysis is a traditional ANOVA on a set of data from page 106 in Kerlinger and Pedhazur. The analysis is straightforward but illustrates an important data entry procedure on the microcomputer. Data should be included within programs as program steps. The entry of data as input on demand from the program means that any errors, or modifications require the entry of the entire set from start to finish. If data is included as part of data statements with the program, modifications can easily be made, errors corrected, and more data added with little difficulty.

So as shown in the following example, the data is for three groups and is listed within the program as lines 500-999. (The shadowed lines are responses to computer prompts.) Note that the data for three groups is indicated by line 500 data 3. The data follows on lines 501, 502, 503.

To modify data, merely retype a line such as 502 data 7, 8, 9, 10, 10, 999. Any subsequent run would reflect this modification. To save modified data, type SAVE (or CSAVE), and the name of the program (in quotes or not depending on the brand of microcomputer you are using).

Some comments on the analysis. In addition to the standard F value, the values of ETA square, Omega square and the probability level are included.

```
]RUN

   - ANALYSIS OF VARIANCE -

      STATISTICAL PROGRAMS

         FOR THE APPLE BY

           CARL F. BERGER
        UNIVERSITY OF MICHIGAN

PRESS <RETURN> TO START
ANALYSIS OF VARIANCE
THIS PROGRAM CALCULATES THE F VALUE
USED TO FIND IF THERE IS A DIFFERENCE
IN THE MEANS OF GROUPS OF DATA.
NEED INSTRUCTIONS?
?YES
TYPE IN YOUR DATA STARTING ON LINE 500. TYPE 500 DATA K
 501 DATA A1,A2,A3,...999
 502 DATA B1,B2,B3,...999 ETC. WHERE K  IS THE NUMBER OF GROUPS AND A, B, ARE
ATAPOINTS
ENTER ONLY 40 NUMBERS PER LINE AND LIST K AS THE FIRST NUMBER ONLY
999 STOPS THE DATA SET FOR EACH GROUP.
  ' HERE'S AN EXAMPLE

500  DATA 3
501  DATA 4,5,6,7,8,999
502  DATA 7,8,9,10,11,999
503  DATA 1,2,3,4,5,999

PRESS <RETURN> TO STARTTHE NUMBER OF GROUPS IS  3
N   MEAN          STANDARD DEV
5   6             1.58113883
5   9             1.58113882
5   3             1.58113883
-------------------------------
UNIVARIATE 1-WAY ANOVA

SOURCE   SUM OF SQ.    D.F.    MEAN SQ.

BETWEEN  90.0000001     2     45.0000001

WITHIN    29.9999998    12    2.49999998

TOTAL    120     14
THE F VALUE IS 18.0000002
WITH 2 AND 12 DEGREES OF FREEDOM
ETA SQUARE= .750000001
OMEGA SQUARE= .693877553
PROBABILITY LEVEL IS 4E-04
```

The subroutines to calculate significance levels and some other subroutines are from Some Basic Statistical Programs.[2]

To introduce an alternate form of analysis that encourages a test for trend, the same data was analyzed using Multiple Linear Regression (MLR) with the following results.

Note similar F values, an R^2 equivalent to ETA^2 and a standard error of the estimate equivalent to the standard deviations of each value (sort of an average standard deviation!).

The means of the data sets are produced by the use of the prediction equation

$$y = 6(X1) + 3(X2) + 6$$

X_1 and X_2 are dummy variables to account for the groups.

Two dummy variables are needed for three groups: a code of 1 and 0 for Group 1, 0 and 1 for Group 2, and 0 and 0 for Group 3 fully cover the relationships.

The use of MLR is preferred by many researchers because of the calculation of the standard error of estimate and the relaxation of problems due to unequal N's.

```
]RUN
```

```
  -LINEAR REGRESSION ANALYSIS-
     STATISTICAL PROGRAMS

        FOR THE APPLE BY

          CARL F. BERGER
       UNIVERSITY OF MICHIGAN
```

```
PRESS <RETURN> TO START


MULTIPLE LINEAR REGRESSION

NEED INSTRUCTIONS?YES
TYPE IN DATA AT LINE 900,
DATA,NUMBER OF INDEPENDENT VAR,SAMPLE SIZE,
EACH INDEPENDENT VARIABLE AND EACH DEPENDENT VARIABLE
HERE IS LINE 900 AS A SAMPLE

900  DATA 2,15,1,0,4,1,0,5,1,0,6,
     1,0,7,1,0,8,0,1,7,0,1,8,0,1,
     9,0,1,10,0,1,11,0,0,1,0,0,2,
     0,0,3,0,0,4,0,0,5

PRESS <RETURN> TO STARTSAMPLE SIZE IS 15
NUMBER OF INDEPENDENT VARIABLES ARE 2
1 0 4
1 0 5
1 0 6
1 0 7
1 0 8
0 1 7
0 1 8
0 1 9
0 1 10
0 1 11
0 0 1
0 0 2
0 0 3
0 0 4
0 0 5

EQUATION COEFFICIENTS:
     CONSTANT:  3
VARIABLE(1):    3
VARIABLE(2):    6

COEFFICIENT OF DETERMINATION(R-SQR)=.75

COEFFICIENT OF MULTIPLE CORRELATION =.866025404

STANDARD ERROR OF ESTIMATE  1.58113883
```

```
F VALUE = 18.0000001
WITH 2 AND 12 DEGREES OF FREEDOM

INTERPOLATION: (ENTER 9999 TO END PROGRAM)
VARIABLE1?1
VARIABLE2?0
DEPENDENT VARIABLE =6.00000001

VARIABLE1?0
VARIABLE2?1
DEPENDENT VARIABLE =9

VARIABLE1?0
VARIABLE2?0
DEPENDENT VARIABLE =3

VARIABLE1?9999
```

Two way analysis of variance is another procedure that can be analyzed on the microcomputer.

Here the data are taken from page 156 of Kerlinger and Pedhazur.

TEACHING METHODS

	Ausabel	Piaget	Skinner
Biology	16 14	20 16	10 14
Chemistry	12 10	17 13	7 7
Physics	7 7	10 8	6 4

A traditional two way analysis often looks like this.

```
PR#0
JRUN
```

```
-TWO WAY ANALYSIS OF VARIANCE-

     STATISTICAL PROGRAMS

      - FOR THE APPLE BY

         CARL F. BERGER
       UNIVERSITY OF MICHIGAN
```

```
PRESS <RETURN> TO CONTINUE

TWO WAY ANALYSIS OF VARIANCE
NEED A INSTRUCTIONS?
?YES
INSERT DATA ON LINE 1080 AS 1080 DATA C,R,X11,...,999,X12,...,999 WHERE C IS THE
NUMBER OF COLUMNS AND R IS THE NUMBER OF ROWS

1080  DATA 3,3
1090  DATA 16,14,999,12,10,999,7,
     7,999
1100  DATA 20,16,999,17,13,999,10
     ,8,999
1120  DATA 10,14,999,7,7,999,4,6,
     999

PRESS <RETURN> TO CONTINUETHE NUMBER OF GROUPS IS. 9
N    MEAN          STANDARD DEV
2    15            1.41421356
2    11            1.41421354
2    7             1.88740797E-04
2    18            2.82842707
2    15            2.82842713
2    9             1.41421355
2    12            2.82842712
2    7             1.88740797E-04
2    5             1.41421356
-----------------------------------
UNIVARIATE 1-WAY ANOVA
SOURCE   SUM OF SQ.     D.F.     MEAN SQ.

BETWEEN   308      8    38.5

WITHIN    31.9999991     9    3.55555545

TOTAL     339.999999     17
::::::::::::::::::::::::::::::::::
THE F VALUE IS 10.8281253
WITH 8 AND 9 DEGREES OF FREEDOM
ETA SQUARE= .905882356
PRESS <RETURN> TO CONTINUETWO WAY ANALYSIS OF VARIANCE

SOURCE OF       SUM OF        DEGREES OF
VARIATION       SQUARES       FREEDOM
```

ROWS	192	2
COLUMNS	108	2
INTERACTION	8.00000024	4
RESIDUAL	31.9999991	9
TOTAL	339.999999	17

VARIANCE ESTIMATE	F VALUE	ETA SQUARE LEVEL
96	27	.318
54	15.188	.565
2.00000006	.563	.024

Notice a very high ETA square of .906 for a univariate ANOVA and the sum of the ETA square levels of .318 + .565 + .024 = .907. Thus the explained variance in the univariate analysis hides the explained variances of the teaching methods, subject matter, and interaction. (As might be expected, the greatest difference is in subject matter, then teaching method, and finally no interaction of teaching method and subject matter.)

Two way analysis of variance can also be analyzed using techniques of Multiple Linear Regression.

The example here is from page 174 of Kerlinger and Pedhazur. First the data must be recast into a form necessary for MLR. Here we will use effect coding.

Dummy Variable

1	2	3	4	5	6	7	8	
1	0	1	0	1	0	0	0	16/14
0	1	1	0	0	0	1	0	12/10
-1	-1	1	0	-1	0	-1	0	7/7
1	0	0	1	0	1	0	0	20/16
0	-1	0	1	0	0	0	1	17/13
-1	-1	0	1	0	-1	0	-1	10/8
1	0	-1	-1	-1	-1	0	0	10/14
0	1	-1	-1	0	0	-1	-1	7/7
-1	-1	-1	-1	1	1	1	1	4/6

	1	2	3
1	16 14	12. 10	7 7
2	20 16	17 13	10 8
3	10 14	7 7	4 6

IMPORTANT INTERACTION

1

.00

1	0	-1
1	0	-1
1	0	-1

Columns

4

.001

0	0	0
1	1	1
-1	-1	-1

Rows

8

.166

0	0	0
0	1	-1
0	-1	1

Interaction

OTHER INTERACTIONS OF INTEREST

2

.005

0	1	-1
0	1	-1
0	1	-1

3

.021

1	1	1
0	0	0
-1	-1	-1

NOT MUCH INTEREST

	5		
	1	0	-1
.716	0	0	0
	-1	0	1

	6		
	0	0	0
.522	1	0	-1
	-1	0	1

	7		
	0	1	-1
.522	0	0	0
	0	-1	1

Again, ETA^2 and R^2 are equivalent in a one way design. In the two way analysis, the rows and columns are represented by two dummy variables. Here a field dummy variable is used, in which 0 and -1 are used so a complete orthogonal effect is obtained.

Using single order correlations, each individual combination can be examined using MLR.

The row ETA^2 equals the 4th single order R^2.

The column ETA^2 equals the 1st single order R^2.

The interaction ETA^2 equals the 8th single order R^2.

In addition, other correlations can be examined for significance such as the second and third single order R^2 values.

Casting the dummy variables in their associated rows and columns allows us to see the relationships. Further, we can start our search for trend.

```
]RUN
::::::::::::::::::::::::::::::::::::::
::::::::::::::::::::::::::::::::::::::
::                                  ::
::  -MULTIPLE LINEAR REGRESSION-    ::
::   - TWO WAY ANALYSIS ADDED  -    ::
::     STATISTICAL PROGRAMS         ::
::                                  ::
::        FOR THE APPLE BY-         ::
::                                  ::
::          CARL F. BERGER          ::
::       UNIVERSITY OF MICHIGAN     ::
::                                  ::
::::::::::::::::::::::::::::::::::::::
::::::::::::::::::::::::::::::::::::::
PRESS <RETURN> TO CONTINUE


MULTIPLE LINEAR REGRESSION

NEED INSTRUCTIONS?YES
TYPE IN DATA AT LINE 900,
DATA,NUMBER OF INDEPENDENT VAR,SAMPLE SIZE,
EACH INDEPENDENT VAR,AND EACHDEPENDENT VARIABLE
HERE IS LINE 900 AS A SAMPLE

900  DATA 8,18,1,0,1,0,1,0,0,0,16
     ,1,0,1,0,1,0,0,0,14

PRESS <RETURN> TO CONTINUESAMPLE SIZE IS 18
NUMBER OF INDEPENDENT VARIABLES ARE 8
1 0 1 0 1 0 0 0 16
1 0 1 0 1 0 0 0 14
0 1 1 0 0 0 1 0 12
0 1 1 0 0 0 1 0 10
-1 -1 1 0 -1 0 -1 0 7
-1 -1 1 0 -1 0 -1 0 7
1 0 0 1 0 1 0 0 20
1 0 0 1 0 1 0 0 16
0 1 0 1 0 0 0 1 17
0 1 0 1 0 0 0 1 13
-1 -1 0 1 0 -1 0 -1 10
-1 -1 0 1 0 -1 0 -1 8
1 0 -1 -1 -1 -1 0 0 10
1 0 -1 -1 -1 -1 0 0 14
0 1 -1 -1 0 0 -1 -1 7
0 1 -1 -1 0 0 -1 -1 7
-1 -1 -1 -1 1 1 1 1 4
-1 -1 -1 -1 1 1 1 1 6
THANKS FOR THE DATA. NOW LETS SEE...
EQUATION COEFFICIENTS:
     CONSTANT:  11
VARIABLE(1):    4
VARIABLE(2):    0
VARIABLE(3):    0
VARIABLE(4):    3
VARIABLE(5):    0
VARIABLE(6):    0
VARIABLE(7):    0
```

```
VARIABLE(8):    1

R-SQUARE=.905882355

MULTIPLE CORRELATION =.951778522

STANDARD ERROR OF ESTIMATE  1.88561806

F VALUE = 10.8281252
WITH 8 AND 9 DEGREES OF FREEDOM
SIGNIFICANCE LEVEL IS 1.3E-03
SINGLE ORDER CORRELATIONS?
PRESS <RETURN> TO CONTINUEVAR#  R-SQR   F    SIG LEVEL
1    .565   .54    0
2    .141   13.5   5E-03
3    .079   7.594   .021
4    .318   30.375   1E-03
5   1E-03   .141    .716
6   6E-03   .563    .522
7   6E-03   .563    .522
8   .024   2.25    .166
PRESS <RETURN> TO CONTINUEINTERCORRELATIONS
1 2  .5
1 3  0
1 4  0
1 5  0
1 6  0
1 7  0
1 8  0
1 9  .751
2 3  0
2 4  0
2 5  0
2 6  0
2 7  0
2 8  0
2 9  .376
3 4  .5
3 5  0
3 6  0
3 7  0
3 8  0
3 9  .282
4 5  0
4 6  0
4 7  0
4 8  0
4 9  .564
5 6  .5
5 7  .5
5 8  .25
5 9  .038
6 7  .25
6 8  .5
6 9  .077
7 8  .5
7 9  .077
8 9  .153
```


USE KERLINGER & PEDHAZUR, 1973

PAGES 175-180 FOR PROPORTIONS OF VARIANCE

Now we turn to data which lend themselves more directly to a search for trend.

First a look at data which show definite trend. The analysis is Nth - correlation analysis. The data set is from page 210 of Kerlinger and Pedhazur.

The first step is a plot of the data which has the following values.

X	Y
2	4
2	6
2	5
4	7
4	10
4	10

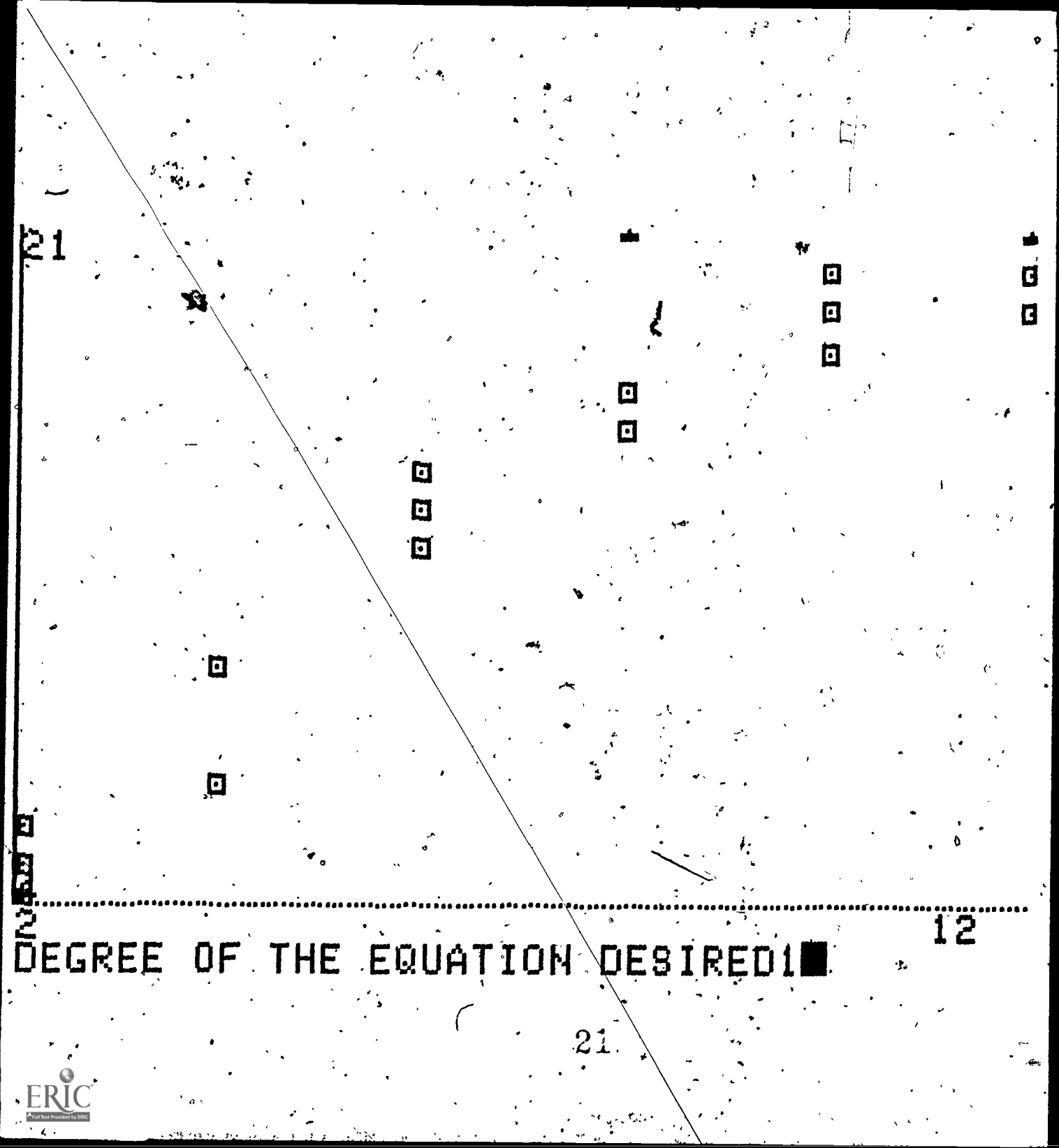
X	Y
6	13
6	14
6	15
8	16
8	17
8	21

X	Y
10	18
10	19
10	20
12	19
12	20
12	21

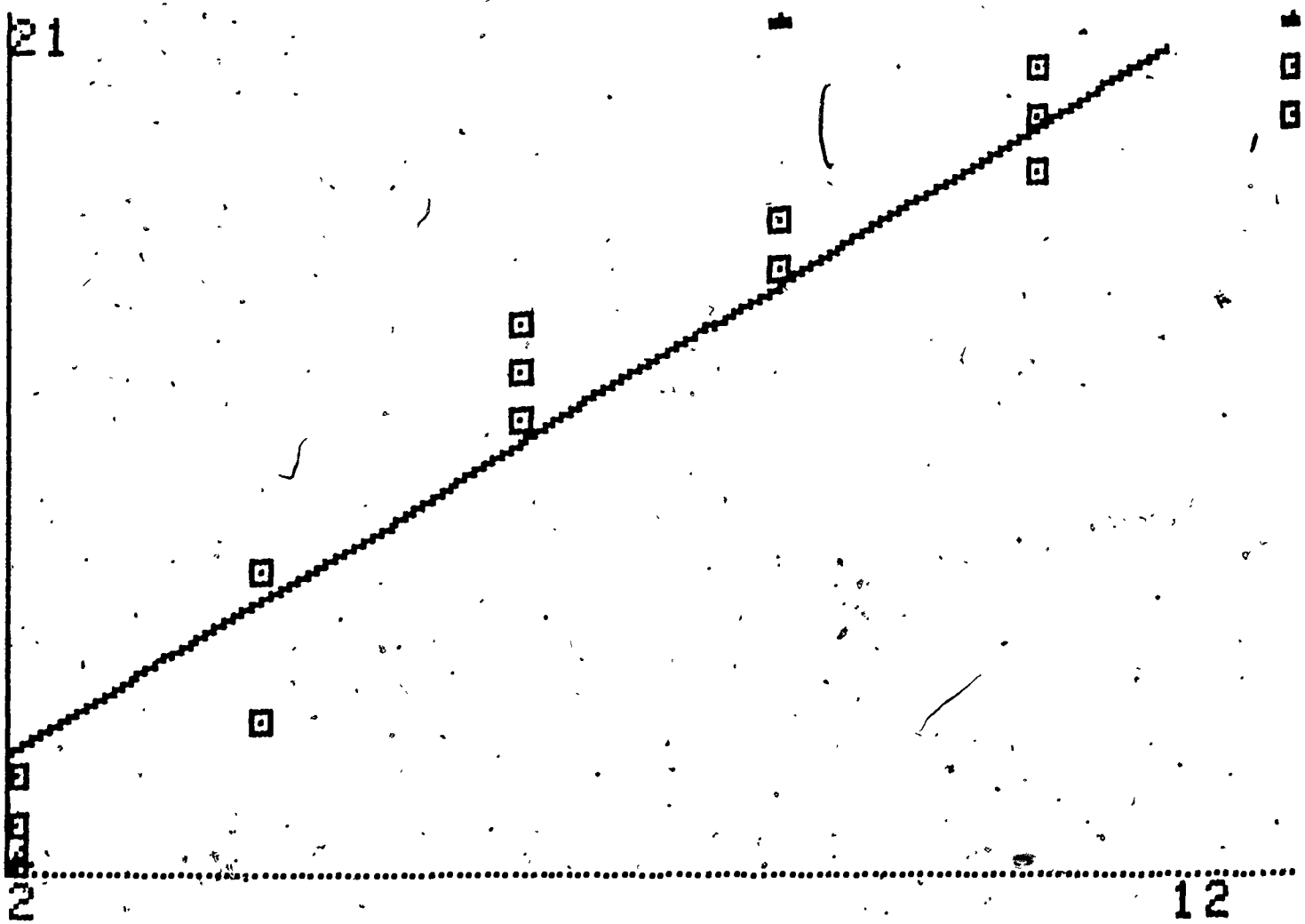
```
]
]RUN
```

```
  -FITTING HIGHER ORDER PLOTS-

  CONTAINS SOME COMMON BASIC SUB
     PROGRAMS BY

        FOR THE APPLE BY
         CARL F. BERGER
     UNIVERSITY OF MICHIGAN
```

```
PRESS <RETURN> TO START

CORRELATION PLOTING
DO YOU NEED INSTRUCTIONS?
?YES

490  REM  *** INSTRUCTIONS ***
500  REM    TYPE IN DATA AS 510 D
     ATA X1,Y1,...999,999
502  REM    WHERE X,Y ARE X VALUES
     & Y VALUES.
504  REM  USE 999,999,999 TO STOP

510  DATA,  2,4,2,6,2,5,4,7,4,10
     ,4,10,6,13, 6,14,6,15,8,16,8
     ,17,8,21,10,18,10,19,10,20,1
     2,19,12,20,12,21
999  DATA 999,999,999

]CONT
```

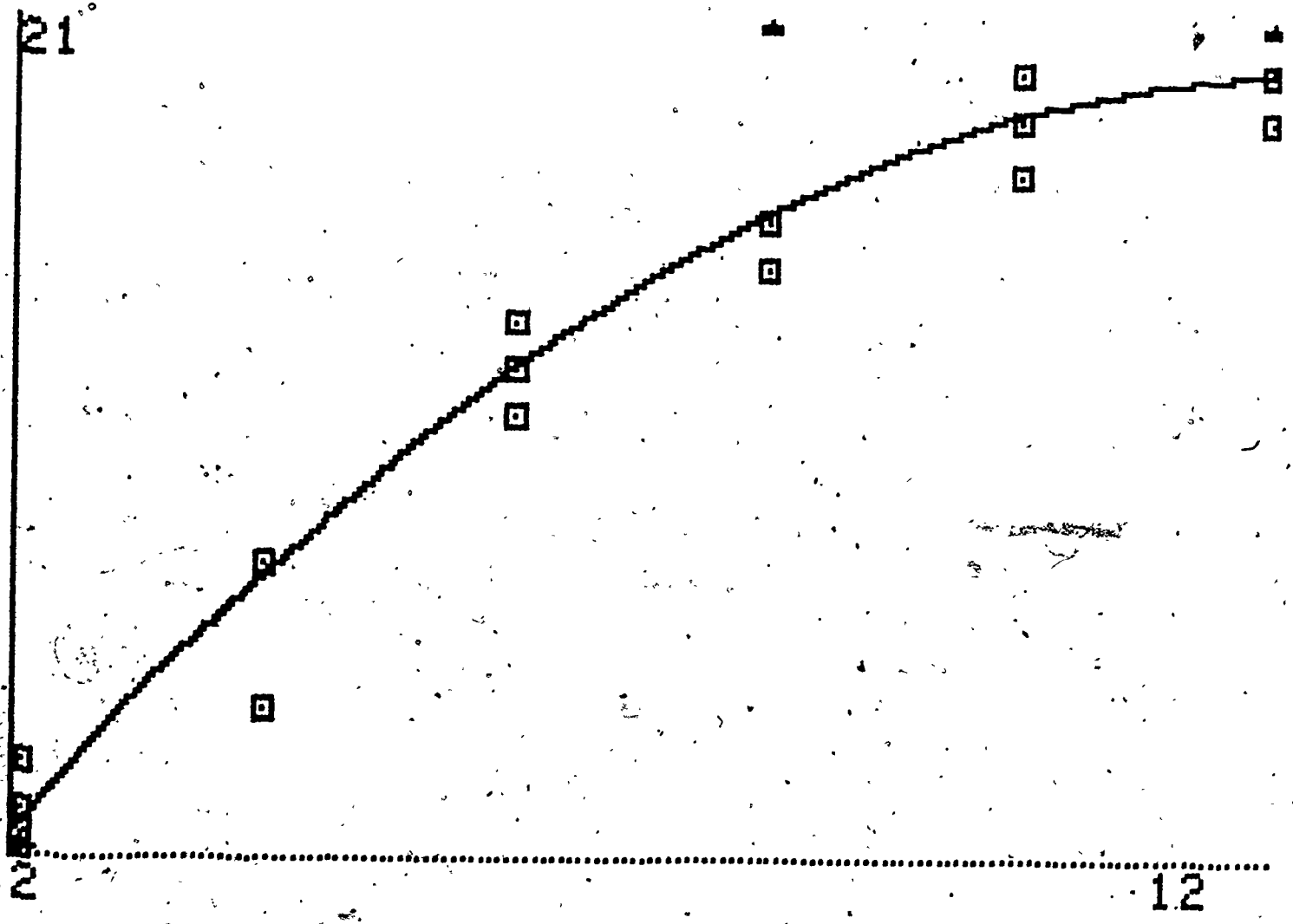

21
12
DEGREE OF THE EQUATION DESIRED1

```
]RUN

  -FITTING HIGHER ORDER PLOTS-

  CONTAINS SOME COMMON BASIC SUB
     PROGRAMS BY

        FOR THE APPLE BY
          CARL F. BERGER
       UNIVERSITY OF MICHIGAN

PRESS <RETURN> TO START

CORRELATION PLOTING
DO YOU NEED INSTRUCTIONS?
?YES

490  REM  *** INSTRUCTIONS ***
500  REM   TYPE IN DATA AS 510 D
     ATA X1,Y1,...999,999
502  REM   WHERE X,Y ARE X VALUES
     , Y VALUES.
504  REM  USE 999,999,999 TO STOP

510  DATA  2,4,2,6,2,5,4,7,4,10
     ,4,10,6,13, 6,14,6,15,8,16,8
     ,17,8,21,10,18,10,19,10,20,1
     2,19,12,20,12,21
999  DATA 999,999,999


]CONT.
DEGREE OF THE EQUATION DESIRED1
THIS MAY TAKE A LITTLE TIME....HMM..
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!

            CONSTANT= 3.26666667
1 DEGREE COEFFICIENT= 1.55714286


COEFFICIENT OF DETERMINATION (R SQR)= .88323628
COEFFICIENT OF CORRELATION= .939806512
STANDARD ERROR OF THE ESTIMATE= 2.05113207

CARE TO LOOK AT THE BEST FIT? N
ANOTHER FIT? Y
```


21
-21-
23
12
ANOTHER FIT? YES

```
?REDIM'D ARRAY ERROR IN 10310
]RUN
::::::::::::::::::::::::::::::::::::
::::::::::::::::::::::::::::::::::::
::                                ::
::  -FITTING HIGHER ORDER PLOTS-  ::
::                                ::
::  CONTAINS SOME COMMON BASIC SUB ::
::     PROGRAMS BY                ::
::                                ::
::         FOR THE APPLE BY       ::
::        CARL F. BERGER          ::
::      UNIVERSITY OF MICHIGAN    ::
::                                ::
::::::::::::::::::::::::::::::::::::
::::::::::::::::::::::::::::::::::::
PRESS <RETURN> TO START

CORRELATION PLOTING
DO YOU NEED INSTRUCTIONS?
?N
DEGREE OF THE EQUATION DESIRED2
THIS MAY TAKE A LITTLE TIME....HMM..
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!

            CONSTANT= -1.90000019
1 DEGREE COEFFICIENT= 3.49464293
2 DEGREE COEFFICIENT= -.138392862

COEFFICIENT OF DETERMINATION (R SQR)= .942770418
COEFFICIENT OF CORRELATION= .970963655
STANDARD ERROR OF THE ESTIMATE= 1.48307909

CARE TO LOOK AT THE BEST FIT? N
ANOTHER FIT? N
```

```
PRESS <RETURN> TO START

CORRELATION PLOTING
DO YOU NEED INSTRUCTIONS?
?N
DEGREE OF THE EQUATION DESIRED3
THIS MAY TAKE A LITTLE TIME....HMM
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!

            CONSTANT= .666666589
1 DEGREE COEFFICIENT= 1.88029105
2 DEGREE COEFFICIENT= .128968252
3 DEGREE COEFFICIENT= -.0127314812


COEFFICIENT OF DETERMINATION (R SQR)= .946268567
COEFFICIENT OF CORRELATION= .972763366
STANDARD ERROR OF THE ESTIMATE= 1.4874756

CARE TO LOOK AT THE BEST FIT? N
ANOTHER FIT? N
```



ANOTHER FIT? YES

```
CORRELATION PLOTING
DO YOU NEED INSTRUCTIONS?
?N
DEGREE OF THE EQUATION DESIRED4
THIS MAY TAKE A LITTLE TIME....HMM..
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
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STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!
STILL THINKING!

            CONSTANT= 8.16669438
1 DEGREE COEFFICIENT= -4.57806223
2 DEGREE COEFFICIENT= 1.8663239
3 DEGREE COEFFICIENT= -.195023529
4 DEGREE COEFFICIENT= 6.51042802E-03


COEFFICIENT OF DETERMINATION (R SQR)= .950915275
COEFFICIENT OF CORRELATION= .975148848
STANDARD ERROR OF THE ESTIMATE= 1.47537072

CARE TO LOOK AT THE BEST FIT? Y
```
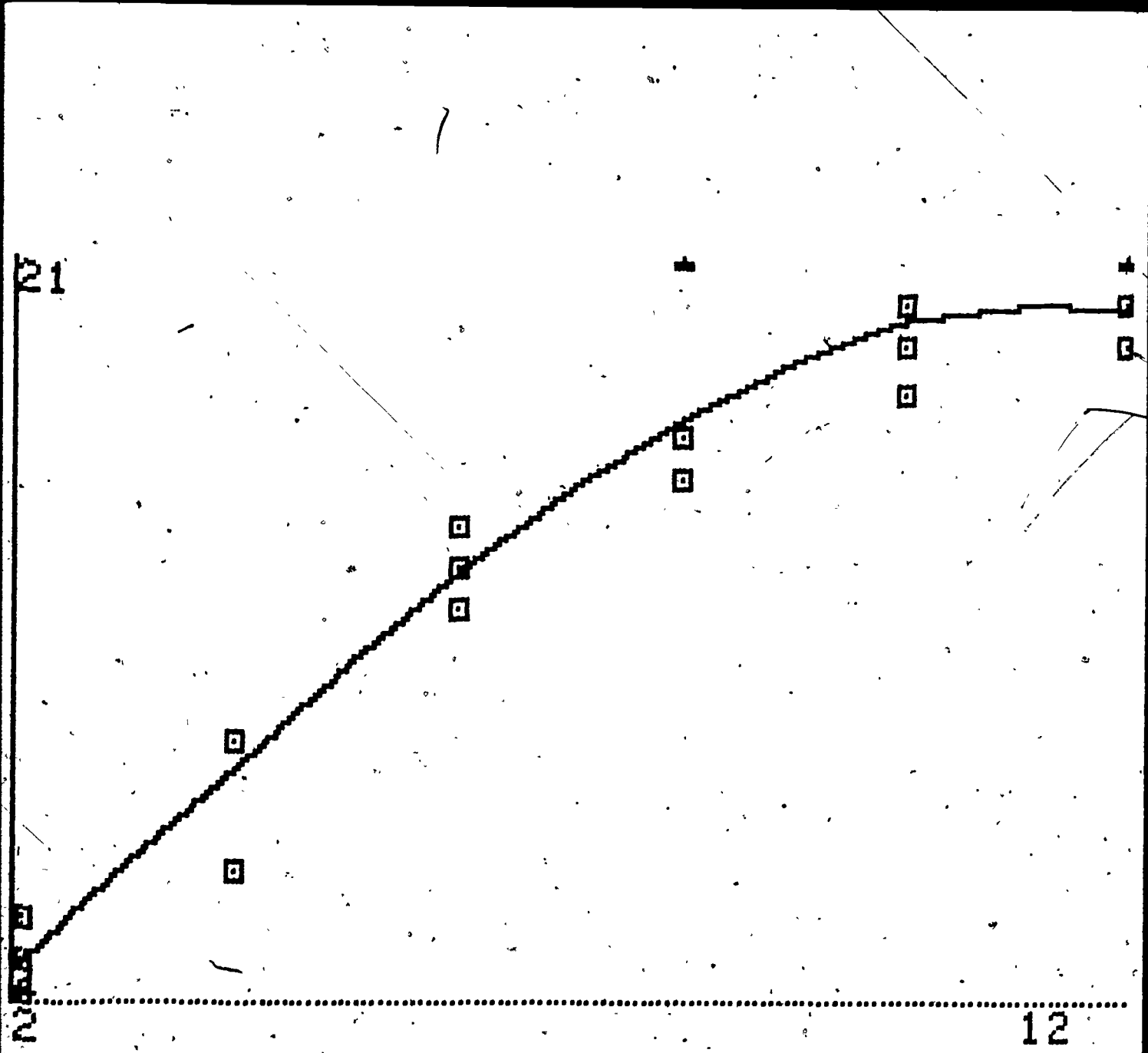
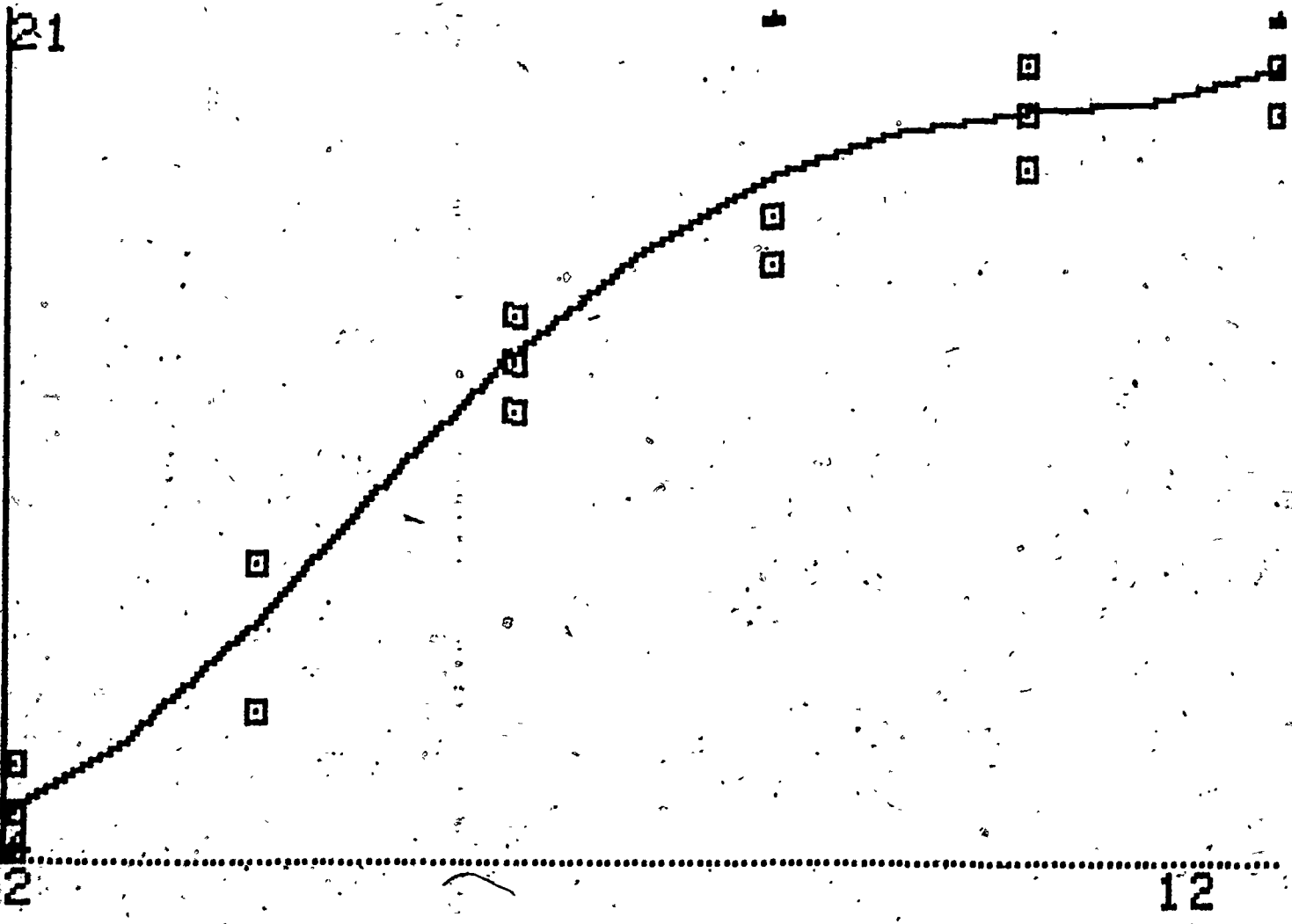
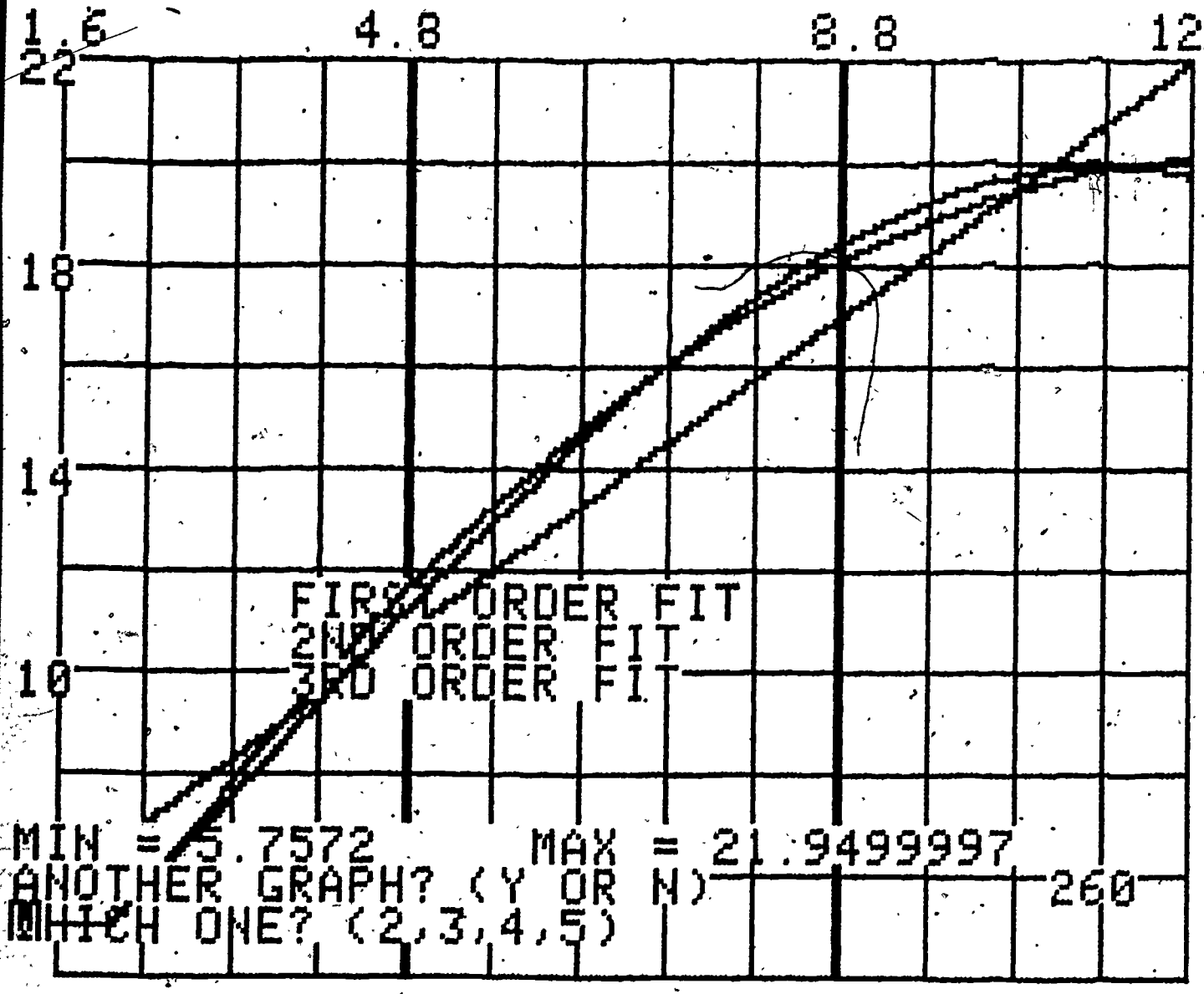



ANOTHER FIT? YES



ANOTHER FIT? NO


1.6
4.8
8.8
12
22
18
14
10
FIRST ORDER FIT
2ND ORDER FIT
3RD ORDER FIT
MIN = 5.7572
MAX = 21.9499997
ANOTHER GRAPH? (Y OR N)
260
WHICH ONE? (2,3,4,5)

Exponential Regression

In addition to higher order polynomial fits (another name for the analysis just completed), we can examine an exponential fit of the form

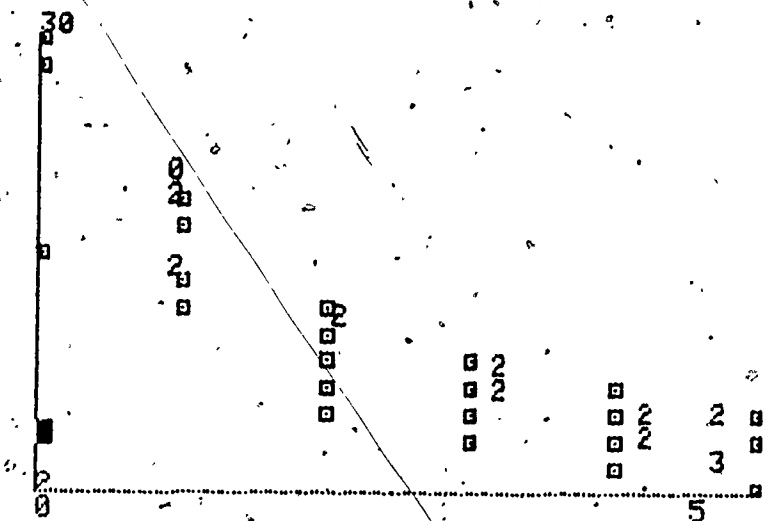
$$y = e^{A} \times e^{BXA}$$

For some rare science education data such fits may work. The example shown is from the findings of students measuring the force and distance between attracting magnets. The data are from six separate student teams.

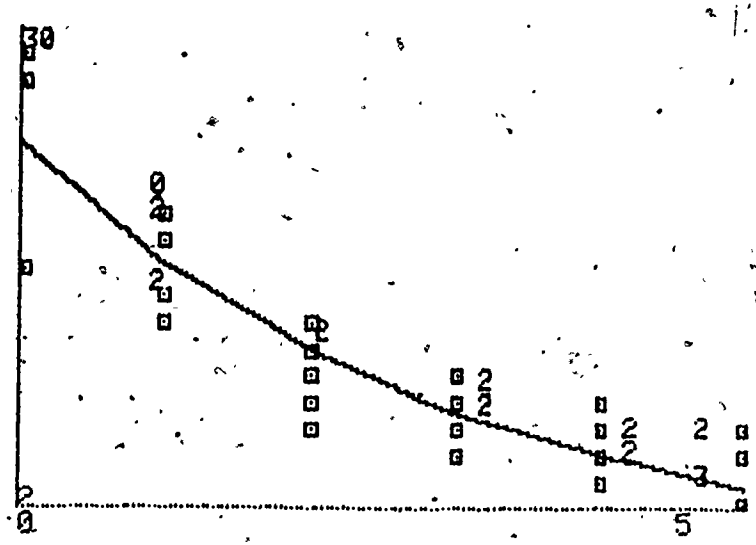
Distance between magnets	Force in washers to release
0	11, 20, 18, 19, 19, 19
1	9, 12, 12, 10, 10, 13
2	5, 6, 6, 7, 8, 8, 9
3	4, 5, 5, 6, 6, 7
4	3, 3, 4, 4, 5, 6
5	2, 2, 2, 4, 4, 4, 5

The computer printout and values are listed on the following page.

```
]
]RUN
::::::::::::::::::::::::::::::::::::
::::::::::::::::::::::::::::::::::::
::                                ::
::  - EXPONENTIAL REGRESSION -    ::
::                                ::
::    STATISTICAL PROGRAMS        ::
::                                ::
::      FOR THE APPLE BY          ::
::                                ::
::        CARL F. BERGER.         ::
::      UNIVERSITY OF MICHIGAN    ::
::                                ::
::::::::::::::::::::::::::::::::::::
::::::::::::::::::::::::::::::::::::
PRESS <RETURN> TO START

CORRELATION PLOTING
DO YOU NEED INSTRUCTIONS?
?YES

490  REM  *** INSTRUCTIONS ***
500  REM  TYPE IN DATA AS 510 DAT
     A X1,Y1,F1,...,999,999,999
502  REM  WHERE X,Y,F ARE X VALUE
     S, Y VALUES, FREQUENCIES.
504  REM  USE 999,999,999 TO STOP

510  DATA 0,11,1,0,20,1,0,18,1,0,
     19,3
520  DATA 1,9,1,1,12,2,1,10,2,1,1
     3,0
530  DATA 2,5,1,2,6,1,2,7,1,2,8,2
     ,2,9,1
540  DATA 3,4,1,3,5,2,3,6,2,3,7,1

550  DATA 4,3,2,4,4,2,4,5,1,4,6,1

560  DATA 5,2,3,5,4,2,5,5,1
999  DATA 999,999,999

EXPONENTIAL REGRESSION


A= 15.7742992
B= -.346410937

COEFFICIENT OF DETERMINATION (R^2)= .849172246
COEFFICIENT OF CORRELATION= .921505424
STANDARD ERROR OF THE ESTIMATE= .25656034


CARE TO LOOK AT THE BEST FIT? Y
20
2
0
```



34



ANOTHER FIT? N

35

Tests for Trend Using Orthogonal and/or Person Vectors

As a final example from Kerlinger and Pedhazur, we use the data set from pages 218 and 219 in a section entitled Trend Analyses with Repeated Measures.

Here not only are there orthogonal variables but also a test for repeated measures--that is, a measure for each person so we can note if the influence of individual differences will make a significant contribution to the explanation of variation of scores.

Note how the data are entered. A small subroutine will generate the dummy variables for the replicated samples and the orthogonal variables are entered as separate data statements starting on line 160.

Having the data in this form saves the massive data entry problems associated with this kind of analysis technique.

```
]RUN
::::::::::::::::::::::::::::::::::::
::::::::::::::::::::::::::::::::::::
::                                ::
::  - REGRESSION ANALYSIS WITH -  ::
::   - REPLICATION OF PERSONS -   ::
::     STATISTICAL PROGRAMS       ::
::                                ::
::        FOR THE APPLE BY        ::
::                                ::
::         CARL F. BERGER         ::
::       UNIVERSITY OF MICHIGAN   ::
::                                ::
::::::::::::::::::::::::::::::::::::
::::::::::::::::::::::::::::::::::::
PRESS <RETURN> TO CONTINUE
REGRESSION ANALYSIS
NEED INSTRUCTIONS? YES
INSERT THE NUMBER OF REPLICATES AT LINE 20
THEN TIMES OR PROBLEMS AT LINE 30
THE CONTRASTS FOR TREND AT LINES 40 AND
AT LINE 160
AND TYPE INDEPENDENT VARIABLES STARTING AT LINE 1550

20 N = 3
25  REM  REPLICATES/SUBJECTS
30 P = 6
35  REM  TIMES/PROBLEMS
40 TT = 5
45  REM  CONTRASTS FOR TREND


160  DATA -5,5,-5,1,-1,-3,-1,7,-3
     ,5


1550  DATA 4,6,5,7,10,10,13,14,15
     ,16,17,21,18,19,20,19,20,21

]RUNPR#1
]
]CONT
SAMPLE SIZE IS 18
-5 5 -5 1 -1 1 0 4
-5 5 -5 1 -1 0 1 6
-5 5 -5 1 -1 0 0 5
-3 -1 7 -3 5 1 0 7
-3 -1 7 -3 5 0 1 10
-3 -1 7 -3 5 0 0 10
-1 -4 4 2 -10 1 0 13
-1 -4 4 2 -10 0 1 14
-1 -4 4 2 -10 0 0 15
1 -4 -4 2 10 1 0 16
1 -4 -4 2 10 0 1 17
1 -4 -4 2 10 0 0 21
3 -1 -7 -3 -5 1 0 18
3 -1 -7 -3 -5 0 1 19
3 -1 -7 -3 -5 0 0 20
```

As the output indicates, the single order correlations can account for each variable separately. But the total shared variation of the single correlations squared do not equal the total R square finally printed out, .9796 versus .9844. The only way to match more closely the total R^2 is to use effect codes and better yet full orthogonal dummy variables.[1]

Nevertheless, the variation can still be isolated and is due to:

1 - a linear component

2 - a quadratic component

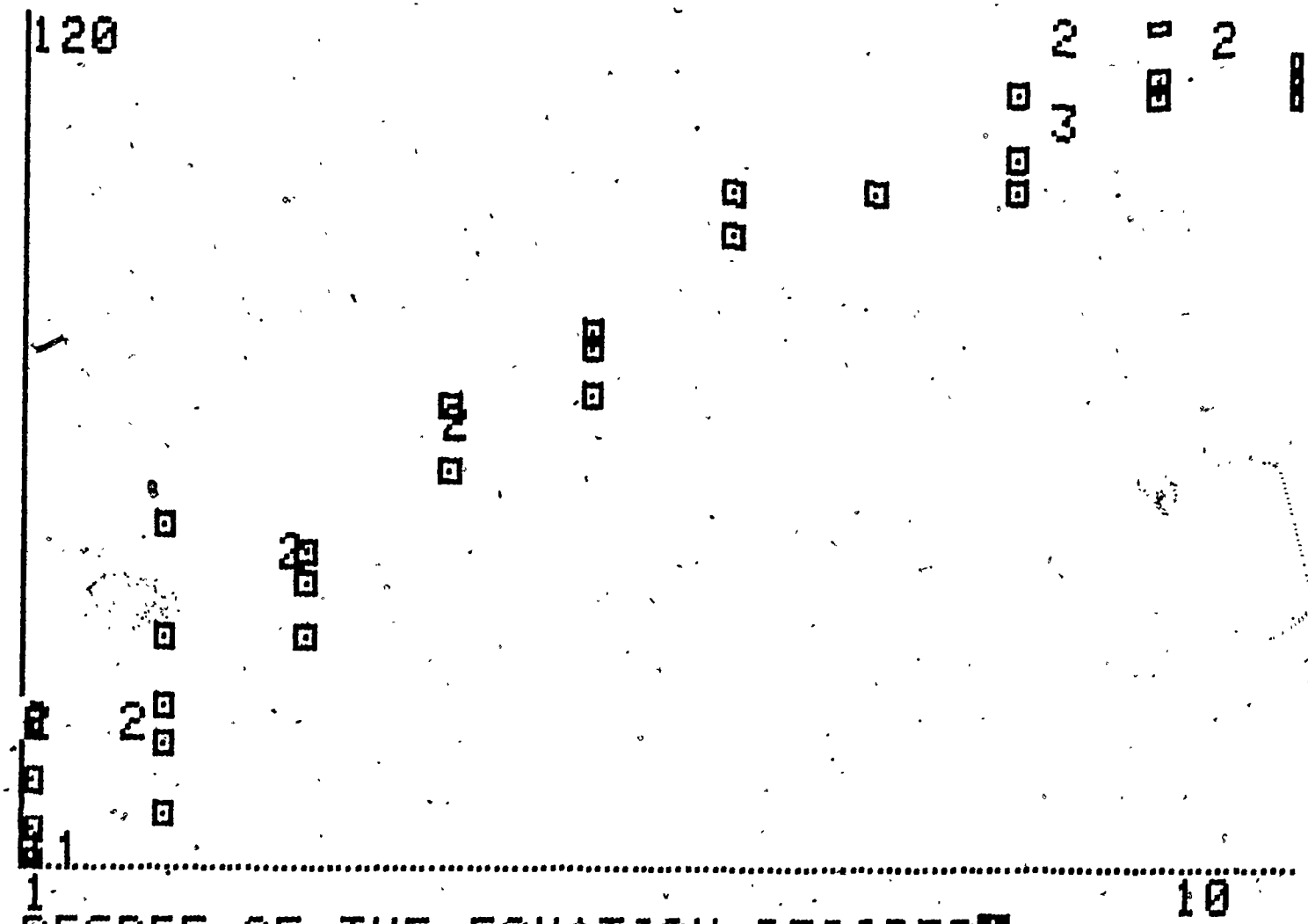
3 - not a cubic component

and some significant individual correlation in variable 6.

```
5 5 5 1 1 1 0 19
5 5 5 1 1 0 1 20
5 5 5 1 1 0 0 21

EQUATION COEFFICIENTS:
     CONSTANT:  15.3333333
VARIABLE(1):    1.55714286
VARIABLE(2):    -.369047619
VARIABLE(3):    -.0611111111
VARIABLE(4):    .178571429
VARIABLE(5):    .0198412698
VARIABLE(6):    -2.5
VARIABLE(7):    -1
PRESS <RETURN> TO CONTINUE
COEFFICIENT OF DETERMINATION(R-SQR)=.984388554

COEFFICIENT OF MULTIPLE CORRELATION =.992163573

STANDARD ERROR OF ESTIMATE  .94868321

F VALUE = 90.0793819
WITH 7 AND 10 DEGREES OF FREEDOM
SIGNIFICANCE LEVEL IS 0


SINGLE ORDER CORRELATIONS?YES
VAR#  R-SQR   F     SIG LEVEL
1     .883    56.76     0
2     .06     38.13     2E-04
3     3E-03    2.24     .1629
4     5E-03    2.98     .1126
5     1E-03     .58     .5833
6     .028    17.78     2.1E-03
7     0        .61      .6144
TOTAL R SQR IS .979618383

LIST1550-

1550  DATA 4,6,5,7,10,10,13,14,15
     ,16,17,21,18,19,20,19,20,21
1560  DATA 101.6,89.6,85.3,106.2,
     112.8,84,73.4,68,108,120
1570  DATA 61.6,91.1,78.8,91.4,94
     ,80.2,72.6,64,63.2,94
```

Some sample problems to test the ideas presented:

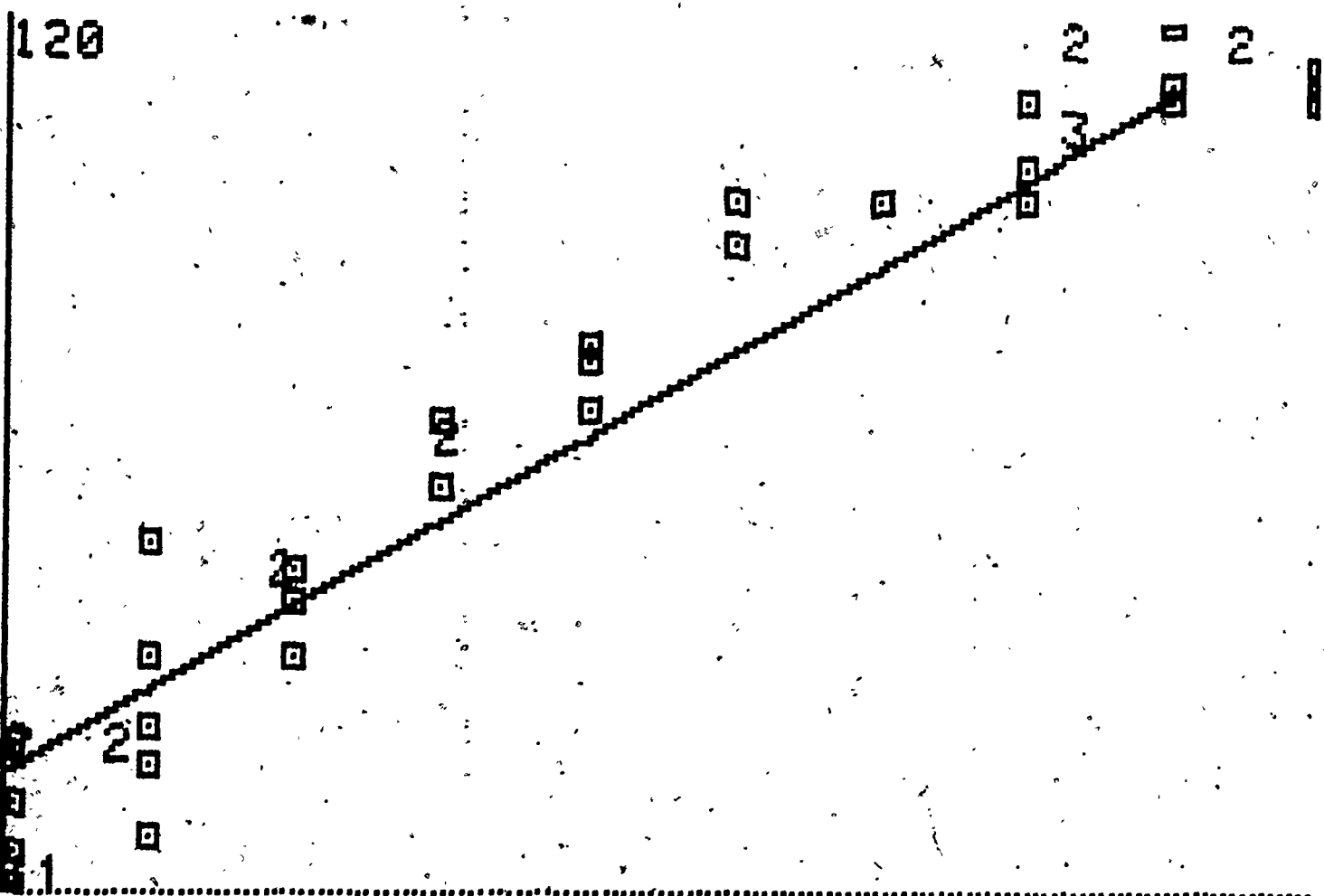
```
                CONSTANT= 15.7927155
1 DEGREE COEFFICIENT= 10.5436623

COEFFICIENT OF DETERMINATION (R SQR)= .9
36587579
COEFFICIENT OF CORRELATION= .96777455
STANDARD ERROR OF THE ESTIMATE= 9.193587
71

CARE TO LOOK AT THE BEST FIT? YES
```



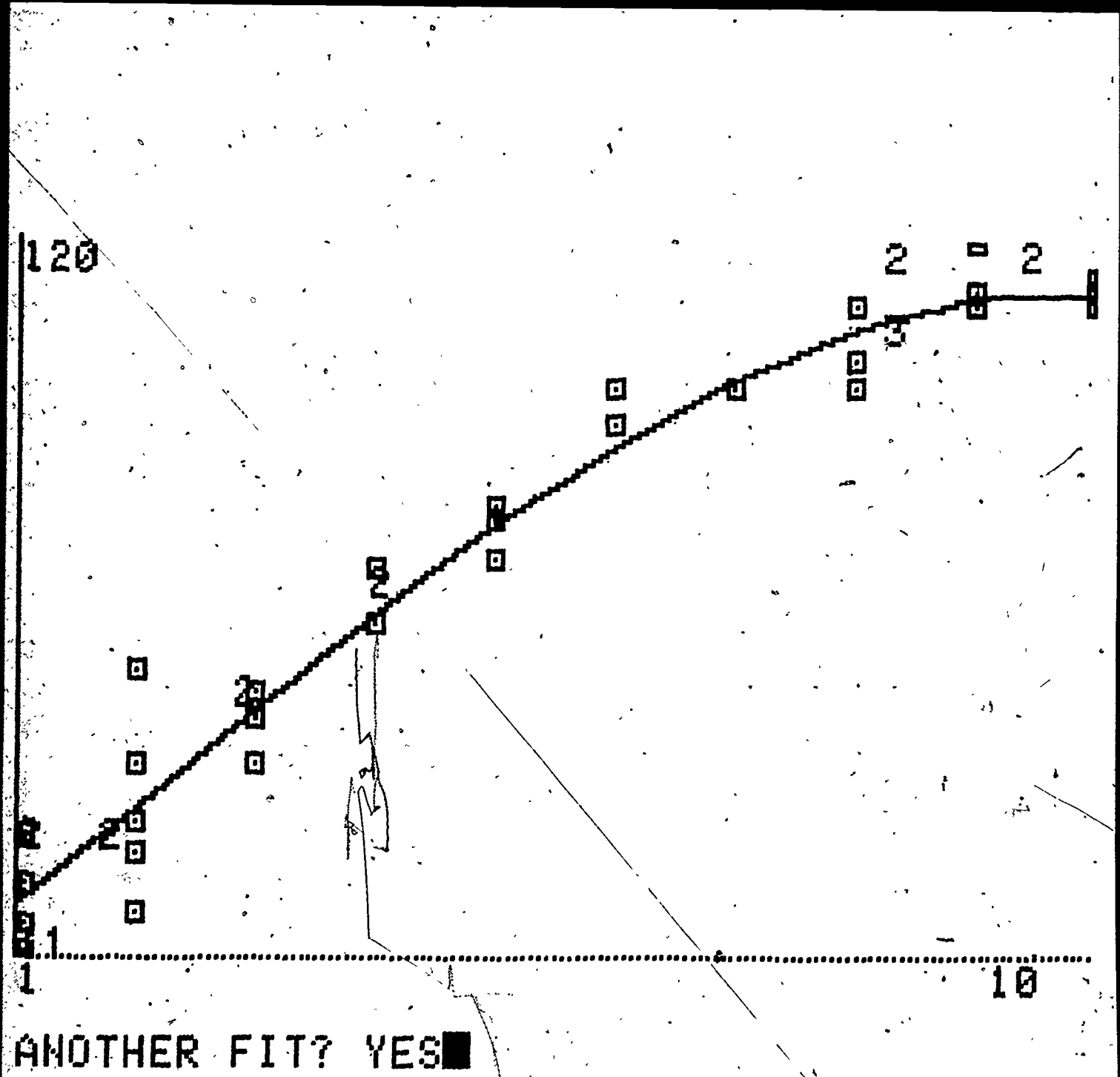
42

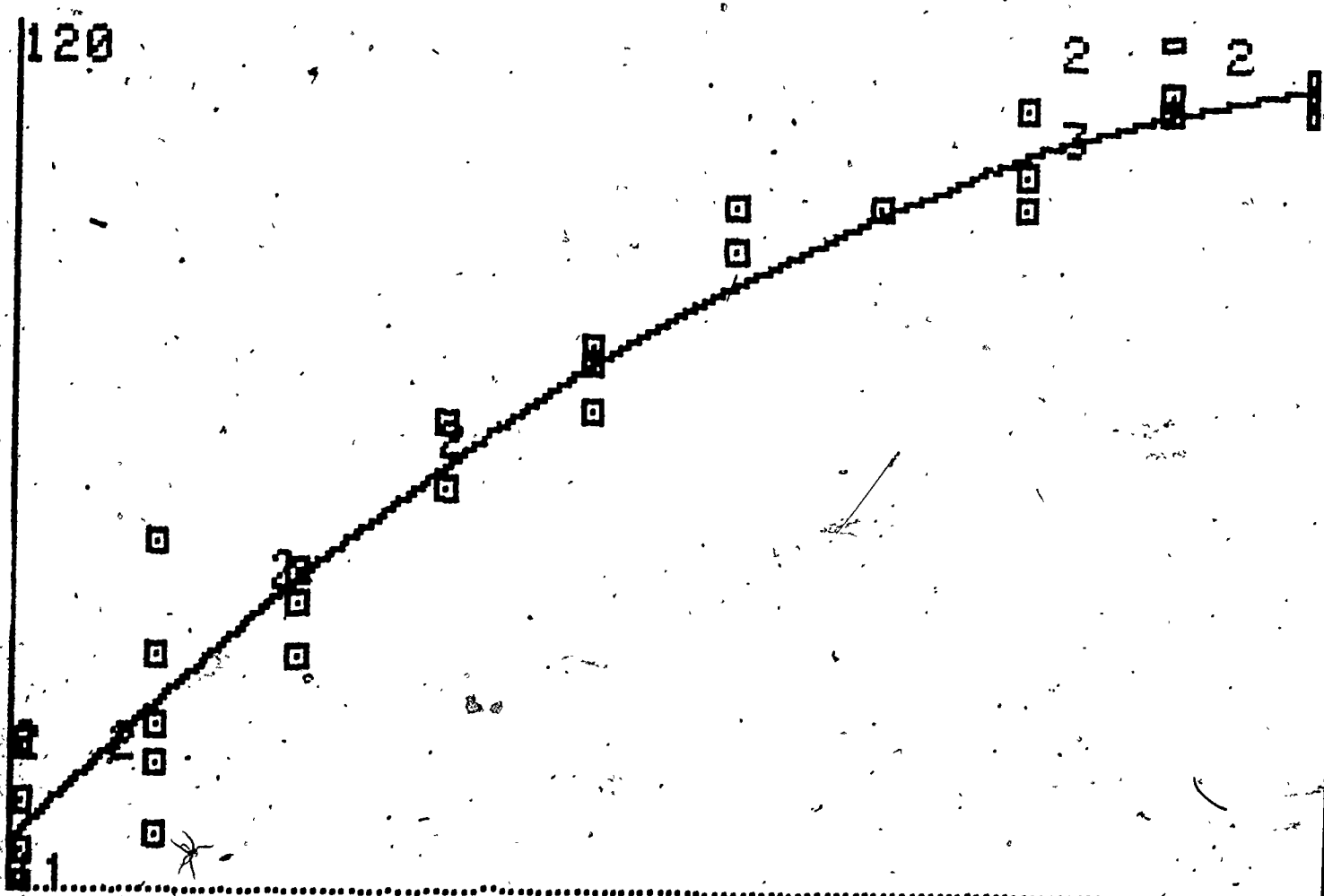
JRUN

43

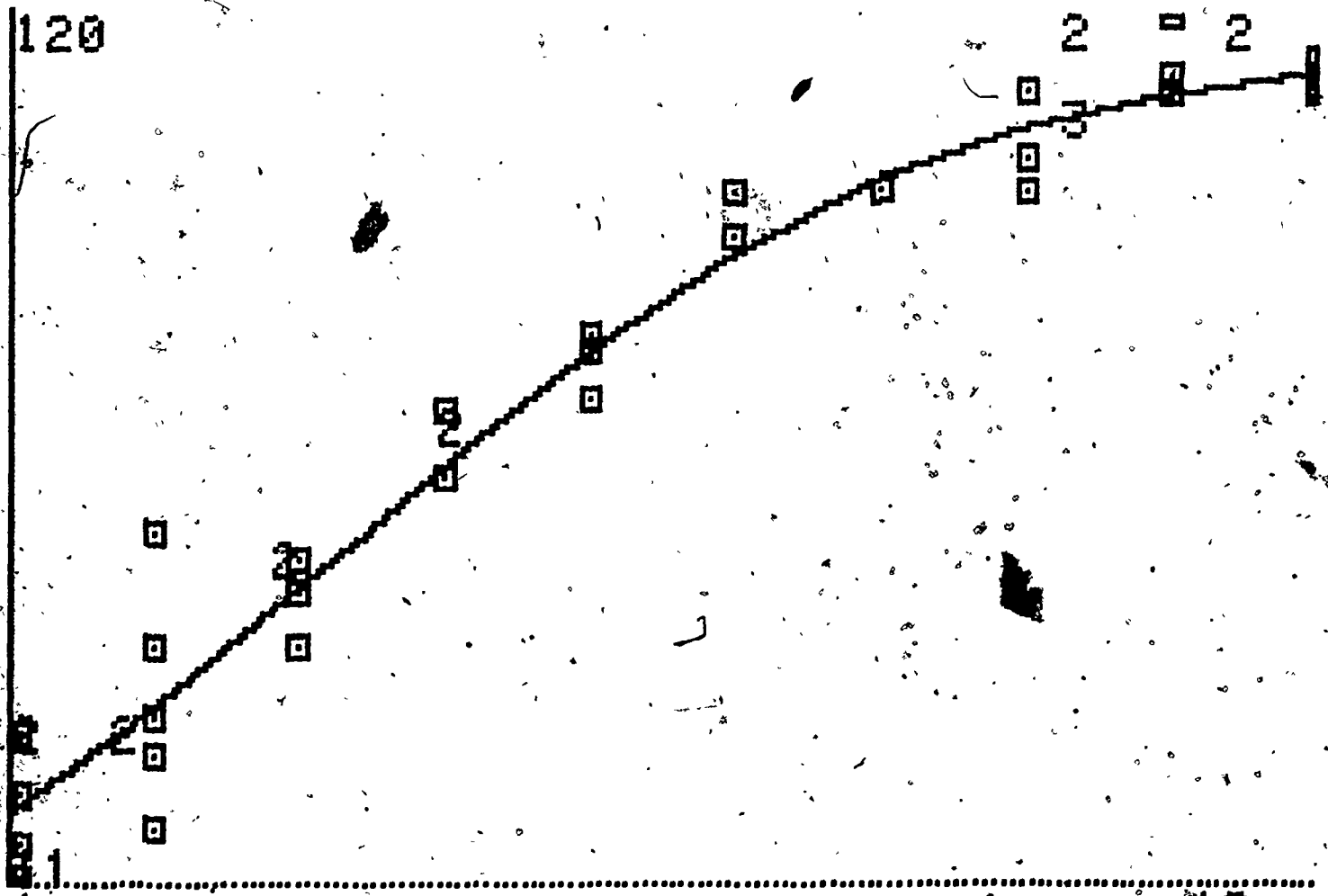


ANOTHER FIT? YES

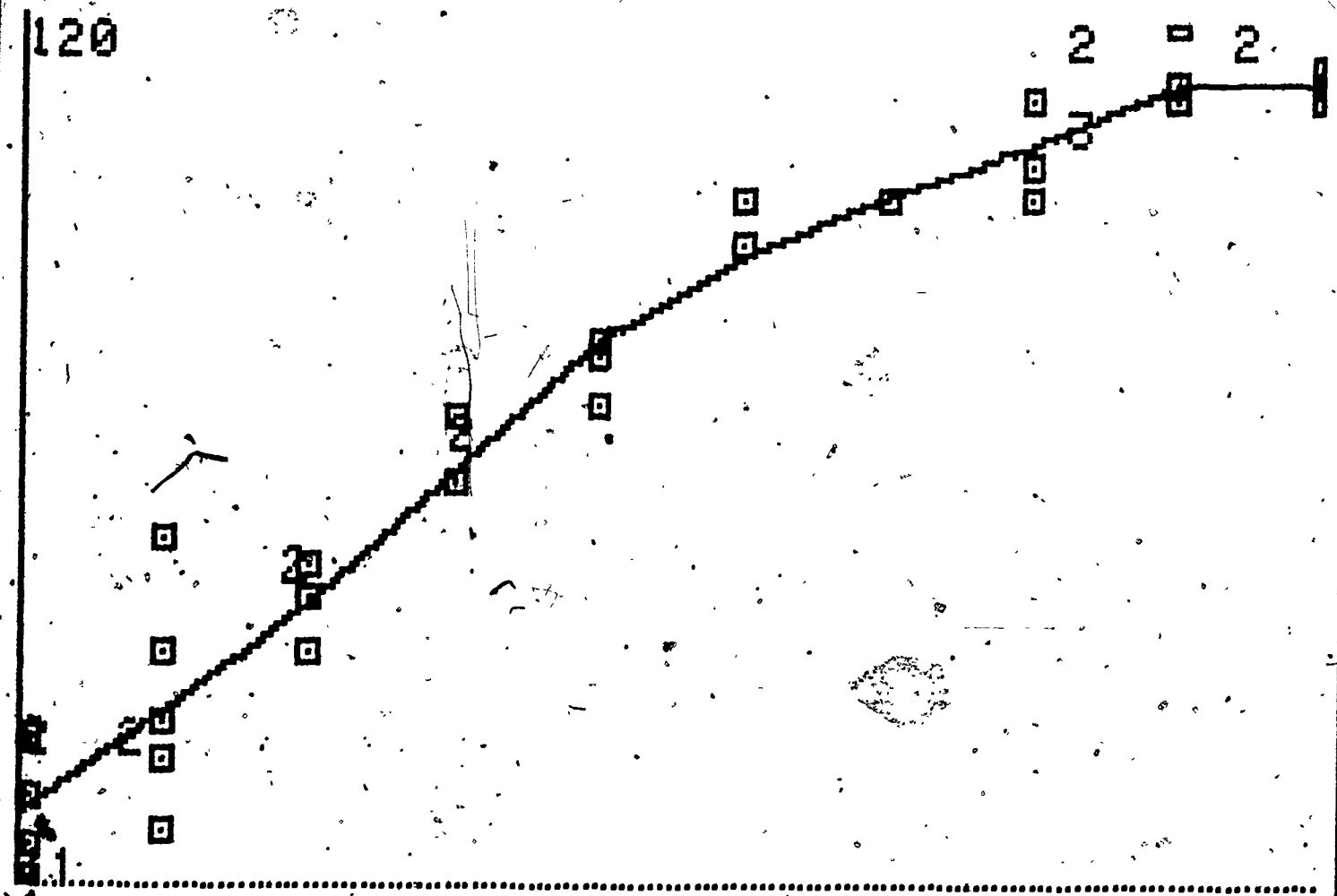

120
2 - 2
1
1
10
ANOTHER FIT? YES



ANOTHER FIT? YES



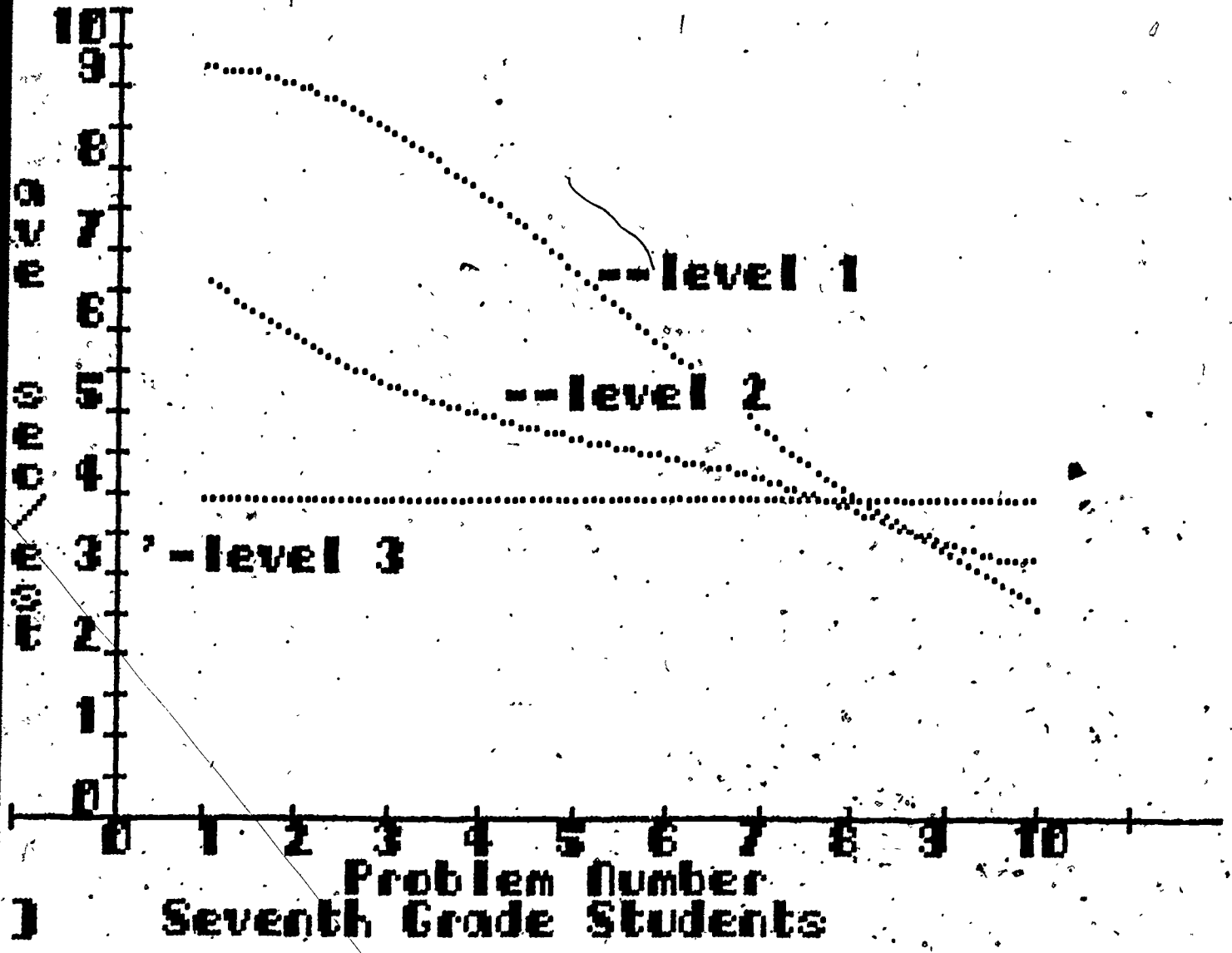
ANOTHER FIT? YES



ANOTHER FIT? ■

```
]
]RUN
REGRESSION ANALYSIS
NEED INSTRUCTIONS? YES
INSERT THE NUMBER OF REPLICATES AT LINE20
THEN TIMES OR PROBLEMS AT LINE 30
THE CONTRASTS FOR TREND AT LINES 40 AND
AT LINE160

20 N = 10
25  REM  REPLICATES/SUBJECTS
30 P = 3: REM  TIMES/PROBLEMS
35  REM  TIMES/PROBLEMS
40 TT = 2: REM  CONTRASTS FOR TRE
     ND
45  REM  CONTRASTS FOR TREND


160  DATA 1,1,0,-2,-1,1

TYPE CONT TO GO ON

BREAK IN 9047
]CONT
SAMPLE SIZE IS 30
1 1 1 0 0 0 0 0 0 0 0 50
1 1 0 1 0 0 0 0 0 0 0 62.2
1 1 0 0 1 0 0 0 0 0 0 101.1
1 1 0 0 0 1 0 0 0 0 0 88
1 1 0 0 0 0 1 0 0 0 0 114.7
1 1 0 0 0 0 0 1 0 0 0 85.9
1 1 0 0 0 0 0 0 1 0 0 62.4
1 1 0 0 0 0 0 0 0 1 0 63.5
1 1 0 0 0 0 0 0 0 0 1 88.6
1 1 0 0 0 0 0 0 0 0 0 115
0 -2 1 0 0 0 0 0 0 0 0 101.6
0 -2 0 1 0 0 0 0 0 0 0 89.6
0 -2 0 0 1 0 0 0 0 0 0 85.3
0 -2 0 0 0 1 0 0 0 0 0 106.2
0 -2 0 0 0 0 1 0 0 0 0 112.8
0 -2 0 0 0 0 0 1 0 0 0 84
0 -2 0 0 0 0 0 0 1 0 0 73.4
0 -2 0 0 0 0 0 0 0 1 0 68
0 -2 0 0 0 0 0 0 0 0 1 108
0 -2 0 0 0 0 0 0 0 0 0 120
-1 1 1 0 0 0 0 0 0 0 0 61.6
-1 1 0 1 0 0 0 0 0 0 0 91.1
-1 1 0 0 1 0 0 0 0 0 0 78.8
-1 1 0 0 0 1 0 0 0 0 0 91.4
-1 1 0 0 0 0 1 0 0 0 0 94
-1 1 0 0 0 0 0 1 0 0 0 80.2
-1 1 0 0 0 0 0 0 1 0 0 72.6
-1 1 0 0 0 0 0 0 0 1 0 64
-1 1 0 0 0 0 0 0 0 0 1 63.2
-1 1 0 0 0 0 0 0 0 0 0 94

EQUATION COEFFICIENTS:
     CONSTANT:  109.666667
VARIABLE( 1):    2.02500001
VARIABLE( 2):    -4.59166667
```

```
VARIABLE( 3):    -38.6000001
VARIABLE( 4):    -28.7000001
VARIABLE( 5):    -21.2666667
VARIABLE( 6):    -14.4666668
VARIABLE( 7):    -2.50000009
VARIABLE( 8):    -26.3000001
VARIABLE( 9):    -40.2000001
VARIABLE(10):    -44.5000001
VARIABLE(11):    -23.0666667

BREAK IN 947
JCONT

COEFFICIENT OF DETERMINATION(R-SQR)=.725080401

COEFFICIENT OF MULTIPLE CORRELATION =.85151653

STANDARD ERROR OF ESTIMATE  14.27963

F VALUE = 4.31578981
WITH 11 AND 18 DEGREES OF FREEDOM
SIGNIFICANCE LEVEL IS 3.4E-03


SINGLE ORDER CORRELATIONS?YES
VAR#  R-SQR    F      SIG LEVEL
 1   7.8981211E-03    .5123     .5123
 2   .121824795    7.97631861    .0109
 3   .0688025652    4.50475768    .0456
 4   7.2123892 4E-03    .5071     .5071
 5   2.32864186E-03    .7021     .7021
 6   .0289307665    1.89420398    .183
 7   .147836582    9.67940625    6.1E-03
 8   1.7577382E-03    .7373     .7373
 9   .0846631703    5.54320999    .0285
10   .135432644    8.86727466    8E-03
11   2.56182574E-04    .8937     .8937
TOTAL R SQR IS .606943597
```


Average Score
10
9
8
7
6
5
4
3
2
1
0
--level 1
--level 2
--level 3
0 1 2 3 4 5 6 7 8 9 10
Problem Number
Seventh Grade Students

REFERENCES AND PROGRAMS

Kerlinger, F.N. and Pedhazur, E.J. Multiple Regression in Behavioral Research. New York: Holt, Rinehart, and Winston, Inc. 1973.

Poole, L., and Borchers, M. Some Common Basic Programs. Berkeley: Osborne/McGraw-Hill, 1978.

```
u
NAME..NTH-CORKP   DATE..03/29/81   TIME..850   PAGE..01
STARTING LINE..0    ENDING LINE..63999

4  GOSUB 50000
5  HOME : PRINT :
10  PRINT : PRINT "CORRELATION PLOTING"
50  PRINT "DO YOU NEED INSTRUCTIONS? "
60  INPUT Z$
70  IF  LEFT$ (Z$,1) = "Y" THEN 480
80  GOTO 10000
480  HOME : LIST 490 - 999: END
490  REM  *** INSTRUCTIONS ***
500  REM    TYPE IN DATA AS 510 DATA X1,Y1,...,999,999
502  REM   WHERE X,Y ARE X VALUES, Y VALUES.
504  REM  USE 999,999,999 TO STOP
510  DATA    2,4,2,6,2,5,4,7,4,10,4,10,6,13, 6,14,6,15,8,16,8,17,8,21,10,1
     8,10,19,10,20,12,19,12,20,12,21
999  DATA 999,999,999
10000  PRINT  CHR$ (4)"BRUN LOMEM:": &  LOMEM: 24576
10010  HGR2 : PRINT  CHR$ (4)"BRUN AS CHR GEN"
10020  HOME
10030  DEF  FN R(X) =  INT (X * 1000 + .5) / 1000
10080  REM *** TO FIND MAX AND MIN TO FILL SCREEN ***
10085  CLEAR
10090  READ X,Y
10100 XI = X:XA = X:YI = Y:YA = Y
10110  READ X,Y
10120  IF X = 999 GOTO 10220
10130  IF X > XI GOTO 10150
10140 XI = X
10150  IF X < XA GOTO 10170
10160 XA = X
10170  IF Y > YI GOTO 10190
10180 YI = Y
10190  IF Y < YA GOTO 10110
10200 YA = Y
10210  GOTO 10110
10220  RESTORE
10230  HOME
10240 XS = 279 / (XA - XI):YS = 159 / (YA - YI)
10250  GOSUB 11170
10260  REM *** NTH-ORDER SUBROUTINE ***
10270  RESTORE
10280  VTAB 22
10290  INPUT "DEGREE OF THE EQUATION DESIRED";D
10300  HGR2
10305  PRINT "THIS MAY TAKE A LITTLE TIME....HMM.."
10310  DIM A(2 * D + 1),R(D + 1,D + 2),T(D + 2)
10320  REM  ENTER DATA   AND POPULATE MATRIX
10330  READ X
10340  IF X = 999 THEN 10480
10350  READ Y
10370 N = N + 1
10380  FOR J = 2 TO 2 * D + 1
10390 A(J) = A(J) + X ↑ (J - 1)
10400  NEXT J
10410  FOR K = 1 TO D + 1
10420 R(K,D + 2) = T(K) + Y * X ↑ (K - 1)
10430 T(K) = T(K) + Y * X ↑ (K - 1)
10440  NEXT K
```

```
NAME..NTH-CORKP   DATE..03/29/81   TIME..850   PAGE..02
STARTING LINE..0    ENDING LINE..63999

10450 T(D + 2) = T(D + 2) + Y * Y
10470  GOTO 10330
10480 A(1) = N
10490  FOR J = 1 TO D + 1
10500  FOR K = 1 TO D + 1
10510 R(J,K) = A(J + K - 1)
10520  NEXT K
10530  NEXT J
10540  FOR J = 1 TO D + 1
10550  FOR K = J TO D + 1
10560  IF R(K,J) < > 0 THEN 10600
10570  NEXT K
10580  PRINT "NON UNIQUE SOLUTION, SORRY!"
10590  GOTO 11090
10600  FOR I = 1 TO D + 2
10610 S = R(J,I)
10620 R(J,I) = R(K,I)
10630 R(K,I) = S
10640  NEXT I
10650 Z = 1 / R(J,J)
10660  FOR I = 1 TO D + 2
10670 R(J,I) = Z * R(J,I)
10680  NEXT I
10690  FOR K = 1 TO D + 1
10695  PRINT "STILL THINKING!"
10700  IF K = J THEN 10750
10710 Z =  - R(K,J)
10720  FOR I = 1 TO D + 2
10730 R(K,I) = R(K,I) + Z * R(J,I)
10740  NEXT I
10750  NEXT K
10760  NEXT J
10765  HGR2
10770  VTAB 1: PRINT
10780  PRINT "          CONSTANT= ";R(1,D + 2)
10790  FOR J = 1 TO D
10800  PRINT J;" DEGREE COEFFICIENT= ";R(J + 1,D + 2)
10810  NEXT J
10820  PRINT
10830 P = 0
10840  FOR J = 2 TO D + 1
10850 P = P + R(J,D + 2) * (T(J) - A(J) * T(1) / N)
10860  NEXT J
10870 Q = T(D + 2) - T(1) ↑ 2 / N
10880 Z = Q - P
10890 I = N - D - 1
10900 J = P / D
10910  PRINT
10920 J = P / Q
10930  PRINT "COEFFICIENT OF DETERMINATION (R SQR.)= ";J
10940  PRINT "COEFFICIENT OF CORRELATION= "; SQR (J)
10950  PRINT "STANDARD ERROR OF THE ESTIMATE= "; SQR ( ABS (Z / I))
10960  PRINT
10970  INPUT "CARE TO LOOK AT THE BEST FIT? ";Z$
10980  IF  LEFT$ (Z$,1) < > "Y" THEN 11090
10990  GOSUB 11170
11000  FOR X = XI TO XA
```

```
NAME..NTH-CORKP   DATE..03/29/81   TIME..850   PAGE..03
STARTING LINE..0    ENDING LINE..63999

11010 P = R(1,D + 2)
11020  FOR J = 1 TO D
11030 P = P + R(J + 1,D + 2) * X ↑ J
11040  NEXT J
11050 Y = P
11060  GOSUB 11110
11070  NEXT X
11080  VTAB 23
11090  INPUT "ANOTHER FIT? ";Z$
11095  IF  LEFT$ (Z$,1) = "Y" THEN 10085
11096  END
11100  LIST 840
11110 PX =  INT (XS * (X - XI)):PY = 159 -  INT (YS * (Y - YI))
11120  HCOLOR= 3
11130  IF PY < 1 THEN 11160
11140  IF PY > 159 THEN 11160
11150  HPLOT EX,EY TO PX,PY
11155 EX = PX:EY = PY
11160  RETURN
11170  REM *** PLOT ROUTINE ***
11180  HGR2 : HCOLOR= 2
11190  VTAB 1: PRINT YA: VTAB 20: HTAB 1: PRINT YI: HTAB 1: PRINT XI;: HTAB
     37: PRINT XA
11200  HPLOT 0,0 TO 0,159 TO 279,159
11210  RESTORE
11220  READ X,Y
11230  IF X = 999 THEN  RETURN
11250 PX =  INT (XS * (X - XI)):PY = 159 -  INT (YS * (Y - YI))
11260  HCOLOR= 3
11270  IF PX = 0 THEN PX = PX + 2
11280  IF PY = 159 THEN PY = PY - 1
11290  HPLOT PX,PY
11300  GOSUB 11390
11320  GOTO 11220
11330 CX =  INT (PX / 6.4):CY =  INT (PY / 8)
11340  IF CX > 38 THEN CX = 38
11350  IF CX = 0 THEN CX = 1
11360  IF CY = 0 THEN CY = 1
11370  HTAB (CX): VTAB (CY): PRINT F
11380  GOTO 11220
11390  REM  *** SQUARE SUBROUTINE ***
11400 XT = PX - 2:XB = PX + 2:YT = PY - 2:YB = PY + 2
11410  IF XT < 1 THEN XT = 1
11420  IF XB > 279 THEN XB = 279
11430  IF YT < 1 THEN YT = 1
11440  IF YB > 159 THEN YB = 159
11450  HPLOT XT,YT TO XT,YB TO XB,YB TO XB,YT TO XT,YT
11460  RETURN
50000  HOME : VTAB 6: PRINT "::::::::::::::::::::::::::::::::::::::::"
50010  PRINT "::::::::::::::::::::::::::::::::::::::::"
50020  PRINT "::                                      ::"
50030  PRINT "::  ";: INVERSE : PRINT " -FITTING HIGHER ORDER PLOTS- ";: NORMAL
     : PRINT "  ::"
50035  PRINT "::                                      ::"
50040  PRINT "::  CONTAINS SOME COMMON BASIC SUB      ::"
50050  PRINT "::     PROGRAMS BY                      ::"
50060  PRINT "::                                      ::"
```

```
NAME..NTH-CORKP   DATE..03/29/81   TIME..850   PAGE..04
STARTING LINE..0    ENDING LINE..63999

50070  PRINT "::          FOR THE APPLE BY          ::"
50080  PRINT "::           CARL F. BERGER           ::"
50090  PRINT "::       UNIVERSITY OF MICHIGAN       ::"
50100  PRINT "::                                    ::"
50110  PRINT "::::::::::::::::::::::::::::::::::::::::"
50120  PRINT "::::::::::::::::::::::::::::::::::::::::"
50130  VTAB 22: INVERSE : PRINT "PRESS <RETURN> TO START";: NORMAL
50140  GET Z$: IF Z$ = " " THEN 50140
50150  RETURN
```

]

** THE LINE CROSS REFERENCE **

PROGRAM NAME...NTH-COR
DATE...03/15/81

480: 10070
490: 480
10000: 10
10085: 11095
10110: 10190 10210
10150: 10130
10170: 10150
10190: 10170
10220: 10120
10330: 10470
10480: 10340
10600: 10560
10750: 10700
11090: 10590 10980
11110: 11060
11160: 11130 11140
11170: 10250 10990
11220: 11320 11380
11390: 11300

** THE VARIABLE CROSS REFERENCE **

PROGRAM NAME...NTH-COR
DATE...03/15/81

A(
10310 10390 10390 10480 10510 10850

CT
10360 10460

CX
11330 11340 11340 11350 11350 11370

CY
11330 11360 11360 11370

D
10290 10310 10310 10310 10310 10380 10410 10420 10450 10450 10490 10500 10540
10550 10600 10660 10690 10720 10780 10790 10800 10840 10850 10870 10890 10900
11010 11020 11030

EX
11150 11155

EY
11150 11155

F
10090 10110 10350 10360 11220 11240 11320 11370

I
10600 10610 10620 10620 10630 10640 10660 10670 10670 10680 10720 10730 107

10730 10740 10890 10950 11240 11310

J
10380 10390 10390 10390 10400 10490 10510 10510 10530 10540 10550 10560 10610
10620 10650 10650 10670 10670 10700 10710 10730 10760 10790 10800 10800 10810
10840 10850 10850 10850 10860 10900 10920 10930 10940 11020 11030 11030 11040

K
10410 10420 10420 10420 10430 10430 10430 10440 10500 10510 10510 10520 10550
10560 10570 10620 10630 10690 10700 10710 10730 10730 10750

N
10370 10370 10480 10850 10870 10890

P
10830 10850 10850 10880 10900 10920 11010 11030 11030 11050

PX
11110 11150 11155 11250 11270 11270 11270 11290 11330 11400 11400

PY
11110 11130 11140 11150 11155 11250 11280 11280 11280 11290 11330 11400 11400

Q
10870 10880 10920

R(
10030 10310 10420 10510 10560 10610 10620 10620 10630 10650 10670 10670 10710
10730 10730 10730 10780 10800 10850 11010 11030

S
10610 10630

T(
10310 10420 10430 10430 10450 10450 10850 10850 10870 10870

X
10030 10030 10090 10100 10100 10110 10120 10130 10140 10150 10160 10330 10340
10390 10420 10430 11000 11030 11070 11110 11220 11230 11250

XA
10100 10150 10160 10240 11000 11190

XB
11400 11420 11420 11450 11450

XI
10100 10130 10140 10240 11000 11110 11190 11250

XS
10240 11110 11250

XT
11400 11410 11410 11450 11450 11450

Y
10090 10100 10100 10110 10170 10180 10190 10200 10350 10420 10430 10450 10450
11050 11110 11220 11250

YA
10100 10190 10200 10240 11190

YB
11400 11440 11440 11450 11450

YI
10100 10170 10180 10240 11110 11190 11250

YS
10240 11110 11250

YT
11400 11430 11430 11450 11450 11450

Z
10650 10670 10710 10730 10880 10950

Z$
10060 10070 10970 10980 11090 11095